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$$\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}, \quad \Delta'=\begin{vmatrix} a' & h' & g' \\ h' & b' & f' \\ g' & f' & c' \end{vmatrix}.$$

$$\Theta=Aa'+Bb'+Cc'+2Ff'+2Gg'+2Hh',$$

$$\Theta'=A'a+B'b+C'c+2F'f+2G'g+2H'h,$$

where A, B, C, etc., are minors of Δ, and A', B', C', etc., are minors of Δ'.

For $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1=0$ and $\dfrac{x^2}{a'^2}+\dfrac{y^2}{b'^2}-1=0$, we find $\Delta'=-\dfrac{1}{a'^2b'^2}$,

$$\Theta=-\frac{1}{a^2\,b'^2}-\frac{1}{a'^2\,b^2}-\frac{1}{a^2\,b^2}, \quad \Theta'=-\frac{1}{a'^2\,b^2}-\frac{1}{a^2\,b'^2}-\frac{1}{a'^2b'^2}.$$

Substituting in (1), we have

$$\left(\frac{a'^2}{a^2}+\frac{b'^2}{b^2}+1\right)^2=4\left(\frac{a'^2}{a^2}+\frac{b'^2}{b^2}+\frac{a'^2b'^2}{a^2\,b^2}\right)$$

from which $\dfrac{a'}{a}\pm\dfrac{b'}{b}=\pm1$.

The only real solution for ellipses is

$$\frac{a'}{a}+\frac{b'}{b}=1.$$

In problem 142,

$$a'=\frac{ab^2}{a^2+b^2},\ b'=\frac{a^2b}{a^2+b^2},\ \text{from which}\ \frac{a'}{a}+\frac{b'}{b}=1.$$

A very excellent demonstration was received from *G. B. M. ZERR*.

CALCULUS.

NOTE ON CENTER OF CURVATURE.

By GEORGE R. DEAN, A. M., Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

The fact, that the point of intersection of two normals which approach each other is neither at infinity nor at the foot of the normals when they become coincident, is not plain to most students. The difficulty may be overcome, in some instances, in the following manner.

Let (x_1, y_1), (x_2, y_2) be two points of the curve ; m_1; m_2 the slope of the tangents at these points. The equations of the normals are then

$$x+m_1y=x_1+m_1y_1,\ x+m_2y=x_2+m_2y_2.$$

It is proposed to find the coördinates of the intersection of these lines when (x_1, y_1) approaches and becomes coincident with (x_2, y_2). Eliminating x between the equations, we have

$$y=y_1+\frac{1+m_2\left(\dfrac{y_1-y_2}{x_1-x_2}\right)}{\dfrac{m_1-m_2}{x_1-x_2}}.$$

Since $\underset{x_1=x_2}{\text{Lim.}}\left(\dfrac{y_1-y_2}{x_1-x_2}\right)=\dfrac{dy_1}{dx_1}$ and $\underset{x_1=x_2}{\text{Lim.}}\left(\dfrac{m_1-m_2}{x_1-x_2}\right)=\dfrac{d^2y_1}{dx_1^2}$, we have

$$y=y_1+\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y}{dx^2}},\ \text{and } x=x_1-\frac{dy_1}{dx_1}\left\{\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y_1}{dx_1^2}}\right\}.$$

105. Proposed by CHARLES C. CROSS, Meridithville, Va.

From all points in a straight line passing through the center of a given circle tangents are drawn to the circle. If the bases and vertices of all the angles thus formed are made to coincide; required the equation of the curve passing through the tangent points.

Solution by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithica, N. Y.

Let the circle be $x^2+y^2=a^2$, and the given line $y=0$. Then the length of the tangent from any point $(x_1, 0)$ on the given line is $\sqrt{(x_1^2-a^2)}$.

Also the slope of the tangent may be calculated from the equation $mx_1\pm a\sqrt{(1+m^2)}=0$, which is obtained by substituting the point $(x_1, 0)$ in the tangent equation $y=mx\pm a\sqrt{(1+m^2)}$. Solving for m, we have

$$m^2=\frac{a^2}{x_1^2-a^2},\ \text{or } m=\pm\frac{a}{\sqrt{(x_1^2-a^2)}}.$$

To determine the required locus, use polar coördinates, with the common vertex of angles as pole and their common base as initial line; the coördinates (r, θ) of the tangent points are then given by the equations

$$\left.\begin{aligned} r&=\sqrt{(x_1^2-a^2)}\\ \tan\theta&=\frac{\pm a}{\sqrt{(x_1^2-a^2)}}\end{aligned}\right\}$$

Hence, the required locus is $r\tan\theta=\pm a$; in Cartesian coördinates this becomes $x^2(y^2-a^2)+y^4=0$.

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.